Technical Comments

Comment on "Relative Motion of a Body about an Orbiting Satellite"

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IN Ref. 1, DeWitt developed a system of linearized equations for the relative motion of a body ejected from a satellite in a circular orbit with small impulse. He used these equations to study the motion of a body ejected in various directions. The linearization procedure requires that the coordinates of the body relative to the satellite, i.e., ξ, η, ζ, be $\ll 1$. If the body is ejected with any tangential component ($\eta_0 \neq 0$), no matter how small, ξ and η will grow to be of order-of-magnitude unity given sufficient time. For example, as the central angle between the satellite and the ejected body approaches $\pi/2$, ξ and η will both approach unity and as this angle approaches π, ξ will approach 2. Therefore, the validity of the solutions is restricted in time as dictated by the secular terms in Eqs. (8) of Ref. 1 when $\eta_0 \neq 0$.

Equations of relative motion have been derived in which the restrictions in DeWitt's analysis are relaxed. Burri[2] has derived such a system of equations in a curvilinear coordinate frame referred to as the "h-s plane." Here, s is the arc length measured from the satellite along its orbit in the direction of motion and h is the altitude above the satellite orbit. The out-of-plane coordinate z is equivalent to $R\zeta$. In terms of h and s, ξ and η are given by

$$\xi = (1 + h/R)\cos(s/R) - 1 \tag{1}$$

and

$$\eta = (1 + h/R)\sin(s/R) \tag{2}$$

The equations of relative motion in the h-s plane can be derived by perturbing Kepler's equations in cylindrical coordinates. Thus,

$$\ddot{h} - (1 + h/R)[n^2R + 2n\dot{s} + R^{-1}\dot{s}^2] = -\mu R^{-2}(1 + h/R)[(1 + h/R)^2 + (z/R)^2]^{-3/2} \tag{3}$$

$$\ddot{s}(1 + h/R) + 2n\dot{h} + 2R^{-1}\dot{s}\dot{h} = 0 \tag{4}$$

and

$$\ddot{z} = -\mu R^{-2}z[(1 + h/R)^2 + (z/R)^2]^{-3/2} \tag{5}$$

where n and μ are those of Ref. 1. Linearization of Eqs. (3–5) requires that h/r and $z/R \ll 1$ and $\dot{s} \ll nR$ which is always true for small ejection rates. There is no restriction on the magnitude of s. The linearized equations are

$$\ddot{h} = 4\pi\dot{s} + 12\pi^2h \tag{6}$$

$$\ddot{s} = -4\pi\dot{h} \tag{7}$$

and

$$\ddot{z} = -4\pi^2z \tag{8}$$

where $R = 1$, $n = 2\pi$, and $\mu = 4\pi^2$.

Equations (6–8) are of a form identical to that of Eqs. (7) of Ref. 1. This is expected because for small η, $\eta \approx s/R$, and $\xi \approx h/R$ as can be shown from Eqs. (2) and (3). The range of validity of the remainder of DeWitt's analysis can be extended if ξ and η are replaced by h/R and s/R, respectively. If the true Cartesian coordinates, ξ and η, are desired they can be calculated from Eqs. (1) and (2).

It is noted that the error in the solution for the secular variable, s, increases with time due to the linearization. However, the characteristics of the motion are preserved in the h-s plane.

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References

[1] DeWitt, R. N., "Relative motion of a body about an orbiting satellite," J. Spacecraft Rockets **3**, 1799–1802 (1966).

[2] Burri, H. U., "A study of certain aspects of lunar ascent and rendezvous with an orbiting vehicle," *2nd Manned Space Flight Meeting* (American Institute of Aeronautics and Astronautics, New York, 1964), pp. 134–139.

Announcement: Change in Style for References in AIAA Publications

The Committee of Engineering Society Editors, of the Engineers Joint Council, has recommended a standard style for references in engineering publications. In the interest of reducing the burden on authors and editors and minimizing confusion, the AIAA Publications Department has decided to follow the recommended style. Examples of the new style will be found below and on the inside back cover of all AIAA journals. The changes will be effective with manuscripts scheduled for the January 1968 issues and thereafter.

Example—Journals

Walker, R. E., Stone, A. R., and Shandor, M., "Secondary Gas Injection in a Conical Rocket Nozzle," *AIAA Journal*, Vol. 1, No. 2, Feb. 1963, pp. 334–338.

Examples—Books

Turner, M. J., Martin, H. C., and Leible, R. C., "Further Development and Applications of Stiffness Method," *Matrix Methods of Structural Analysis*, 1st ed., Vol. 1, Macmillan, New York, 1964, pp. 203–266.

Segrè, E., ed., *Experimental Nuclear Physics*, 1st ed., Vol. 1, Wiley, New York, 1953, pp. 6–10.

Example—Reports

Book, E. and Bratman, H., "Using Compilers to Build Compilers," SP-176, Aug. 1960, Systems Development Corp., Santa Monica, Calif.

Example—Transactions or Proceedings

Soo, S. L., "Boundary Layer Motion of a Gas-Solid Suspension," *Proceedings of the Symposium on Interaction between Fluids and Particles*, Institute of Chemical Engineers, Vol. 1, 1962, pp. 50–63.